RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2016

SECOND YEAR [BATCH 2015-18]

Date : 12/12/2016

COMPUTER SCIENCE [Honours] Paper : III

Time : 11 am – 3 pm

Full Marks : 75

[Use a separate Answer Book for each Group]

$\underline{Group} - \underline{A}$

swer	r <u>any one</u> question:-	[1×5]
a) b)	Give an application of Principle of Inclusion and Exclusion. Three electric bulbs are chosen at random from 15 bulbs, of which 5 are defective. Find the	
		[21/2]
a)		[21/2+21/2]
swer	any two questions:-	[2×10]
a)	If A, B, C are non-empty sets, then prove that $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$.	[3]
b)	Let <i>R</i> be the relation on the act $S = \{2, 3, 4, 5\}$, defined by <i>xRy</i> when $ x - y $ is divisible by 3.	
``		[4]
c)	Examine whether the mapping $f: R \to R$ defined by $f(x) = ax + b$, $a \neq 0$ is bijective or not.	[3]
a)	What do you mean by Lexicographic order? Explain with example.	[2]
b)	Let D_n denote the set of all positive divisors of <i>n</i> . Draw the Hasse diagram for the divisibility relation on the set D_{22} . Also check whether it forms lattice or not	[3]
c)		
-)	$x \ge 1, y \ge 2$ and $z \ge 3$?	[2]
d)	Let A be some fixed 10-element subset of $\{1, 2, 3, \dots, 50\}$. Show that A possesses two	
	different 15-element subsets, the sum of whose elements are equal.	[3]
a)	In the Tower of Hanoi Puzzle, suppose our goal is to transfer all n disks from peg 1 to peg 3, but we can't move a disk directly between peg 1 and peg 2, each move of a disk must be	
	the puzzle for <i>n</i> disks, with initial condition.	[5]
b)		
	(ii) A book contains 100 misprints distributed randomly throughout it's 100 pages. What is the probability that a page observed at random contains at least 2 misprints.	[2+3]
a)	If a, b are any two elements of a group $(G,*)$ then show that the equations $a*x=b$ and	l
		[3]
b)	Prove that every field is an integral domain.	[3]
c)	Prove that the set {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition modulo 5 as composition.	[4]
	 a) b) a) swer a) b) c) a) b) c) d) a) b) a) 	 b) Three electric bulbs are chosen at random from 15 bulbs, of which 5 are defective. Find the probability that at least one is defective. a) Explain with example the following two terms : i) Cyclic group ii) Pigeon Hole Principle. a) If <i>A</i>, <i>B</i>, <i>C</i> are non-empty sets, then prove that (<i>A</i>−<i>B</i>)∪(<i>B</i>−<i>A</i>) = (<i>A</i>∪<i>B</i>)−(<i>A</i>∩<i>B</i>). b) Let <i>R</i> be the relation on the act <i>S</i> = {2, 3, 4, 5}, defined by <i>xR</i> when <i>x</i>−<i>y</i> is divisible by 3. Write <i>R</i> as a set of order pairs and examine whether it is an equivalence relation or not. c) Examine whether the mapping <i>f</i>: <i>R</i>→<i>R</i> defined by <i>f</i>(<i>x</i>) = <i>ax</i>+<i>b</i>, <i>a</i>≠0 is bijective or not. a) What do you mean by Lexicographic order? Explain with example. b) Let <i>D_a</i> denote the set of all positive divisors of <i>n</i>. Draw the Hasse diagram for the divisibility relation on the set <i>D</i>₃₀. Also check whether it forms lattice or not. c) How many solutions are there of <i>x</i>+<i>y</i>+<i>z</i>=17, subject to the constraints <i>x</i>≥1, <i>y</i>≥ 2 and <i>z</i>≥3? d) Let <i>A</i> be some fixed 10-element subset of {1, 2, 3,, 50}. Show that <i>A</i> possesses two different 15-element subsets, the sum of whose elements are equal. a) In the Tower of Hanoi Puzzle, suppose our goal is to transfer all <i>n</i> disks from peg 1 to peg 3, but we can't move a disk directly between peg 1 and peg 3, each move of a disk must be a move involving peg 2. Find a recurrence relation for the number of moves required to solve the puzzle for <i>n</i> disks, with initial condition. b) (i) State Poisson distribution. (ii) A book contains 100 misprints distributed randomly throughout it's 100 pages. What is the probability that a page observed at random contains at least 2 misprints. a) If <i>a</i>, <i>b</i> are any two elements of a group (<i>G</i>,*) then show that the equations <i>a*x</i>=<i>b</i> and <i>y*a</i>=<i>b</i> have a unique solution in <i>G</i>. b) Prove that every field is an integral domain. c

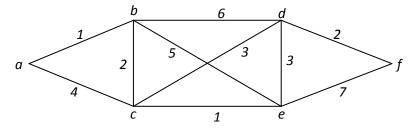
Gro<u>up – B</u>

Answer any one question:-

- a) Does there exist a simple graph with seven vertices having degrees (1, 3, 3, 4, 5, 6, 6)? 7. Explain the reason. $[2\frac{1}{2}]$
 - b) Prove that any two simple connected graphs with *n* vertices, all of degree two, are isomorphic. $[2\frac{1}{2}]$
- What are the two types of Kuratowski's graph? Comment on the planarities of these graphs. $[2\frac{1}{2}+2\frac{1}{2}]$ 8.

Answer any one question:-

- Prove that a non-empty connected graph G is Eulerian if and only if all of it's vertices are of 9. a) even degree.
 - b) Find the shortest path from a to f using Dijkstra's algorithm from following graph:



- c) Prove that there are $\frac{(n-1)}{2}$ edge-disjoint Hamiltonian circuits in a complete graph with n vertices, where n is an odd integer more than or equal to 3. [3]
- 10. a) If a connected planar graph G has n vertices, e edges and r regions, then prove that n-e+r=2. [3] b) What is a cut-set? Prove that every circuit has an even number of edges in common with any
 - cut-set. [1+4]What are the different graph representation techniques? Explain. c)

Group – C

Answer any one question:-[1×5] What do you mean by 'object delegation'? 11. a) [2] Describe the concept of local class with the help of an example? What are the restrictions b) imposed on it? [2+1] Given a member function for a class with four (4) arguments. Say the second argument from 12. a) left takes default value. Will the program execute correctly? Justify with an example in support of your answer. $[2\frac{1}{2}]$ b) Discuss different scenarios where ambiguity occurs during function overloading. $[2\frac{1}{2}]$ Answer any one question:-[1×10] Describe the significance of using 'virtual destructor' with suitable example. [3] 13. a) What do you mean by pure abstract class? Why does C++ not allow to create the object of an b) abstract class? [2+3]

"During binary operator overloading we need to send one argument, whereas need to send c) two arguments while doing the same thing using friend function" – Justify. [2]

[1×5]

[1×10]

[3]

[4]

- [2]

14. a)	Can you catch a base class exception using its derived class object? Justify with an example.	[2]
b)	Differentiate between function overloading and function overriding?	[2]
c)	Explain the concept of static object with an example.	[2]
d)	"Does friend function violate the encapsulation property of a class?" – Justify.	[2]
e)	Explain the concept of overloading function template with an example.	[2]

<u>Group – D</u>

(Answer <u>any two</u> questions) [2×10]

- 15. a) Prove that a binary tree of height H has at most 2^(H+1) -1 nodes. (H>=0). [2]
 b) Inorder traversal of a binary tree uses a stack. How to make this traversal without a stack? Clearly explain the steps in detail. [6]
 c) Consider a Max Heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4. Now consider that the value 35 is inserted into this heap. Heapify the array and show each step during the process. [2]
- 16. a) Give the algorithm of linear probing for collision resolution. What are the disadvantages of this method? Give the criterion for choosing the 2^{nd} hash function in double hashing. [2+1+2]
 - b) Suppose we have a file containing a total of 10K (10,000) characters. The file contains six different characters, and the different characters appear with the following frequencies:

	А	В	С	D	E	F
Frequencies in 100s	5	9	45	12	13	16

Use Huffman coding technique to compress the data. If one character needs one byte space, then compare the space consumption of the compressed and uncompressed file.

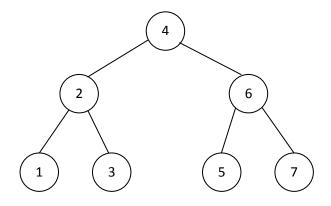
17. a) Consider the following code snippet in C. The function **print**() receives the root of a Binary Search Tree (BST) and a positive integer k as arguments.

```
/* BinarySearchTree node */
struct node {
    int data;
    struct node *left, *right;
};
int count = 0;
void print(struct node *root, int k){
    if (root != NULL && count <= k){
        print(root->right, k);
        count++;
        if (count == k)
            printf("%d ", root->data);
        print(root->left, k);
    }
}
```

[5]

[5]

- I. What is the output of *print(root, 3)* where 'root' represent root of the binary search tree of following figure?
- II. In general, what is the function doing?



	b)	Construct an AVL tree, showing the steps clearly, for the elements that come in the following	
		order:	[3]
		19, 18, 21, 17, 12, 15, 2, 5, 7, 11, 6	
	c)	Define a Red Black Tree. Briefly discuss with an example.	[2]
18.	a)	Distinguish between B-tree and B^+ tree.	[3]
	b)	You are supplied with the postorder traversal of a given Binary Search Tree (BST). Is it possible to construct the BST uniquely? Justify.	[2]
	c)	Consider the following input sequence:	[5]
		Tomato, Lemon, Apple, Honeyberry, Apricot, Papaya, Avocado, Grapefruit,	
		Blueberry, Cherry, Coconut, Cucumber, Pineapple, Grape, Guava, Loquat,	
		Watermelon, Banana, Raspberry, Strawberry.	

Insert the input strings into a hash table of size 26 using the following hash function: H(String) = Fourth letter of String

For example, if the word 'fruit' is to be inserted, then it will be placed at ninth index of the hash table as 'i', the fourth letter of 'fruit', is the ninth letter in the English alphabet.

Show the hash table after each insertion. As a collision resolution technique, use coalesced chaining. Finally report the number of collisions occurred during the insertion.

_____ × _____